

24 TILES

Suggested Grade

4th

SD Mathematics Strand & Standard (*Primary for Task*)

Algebra

4.A.1.2 Students are able to recognize and use the commutative property of addition and multiplication.

Task Summary

Students explore the commutative property by constructing rectangular regions using a set of one-inch tiles.

Time and Context of Task

1 period - 30 to 45 minutes

Materials Needed

24 tiles (1-inch square), paper, and pencil, chart for students (included)

Author and Lead Teacher for the Task

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Chamberlain Elementary School

*The lesson was adapted from the book, Principles and Standards for School Mathematics,
The National Council of Teachers of Mathematics, Inc. – Copyright 2000*

24 TILES

Working with a partner show all the rectangular regions you can make using 24 tiles (1-inch squares). You need to use all the tiles. Count and keep a record of the area and perimeter of each rectangle and then look for and describe any relationships you notice.

Procedure:

1. Working with a partner, students will show all the rectangular regions they can make using 24 tiles. Count and keep a record of the area and perimeter of each rectangle.

Length (L) (units)	Width (W) (units)	Area (A) (sq. units)	Perimeter (P) (units)
1	24	24	50
2	12	24	28
3	8	24	22
4	6	24	20
6	4	24	20
8	3	24	22
12	2	24	28
24	1	24	50

2. When the students are ready to discuss their results, ask if anyone has a rectangle with a length of 1, of 2, of 3, and so on, and has a model of the way they organized the information.
3. Ask if anyone tried to form a rectangle of length 5 and, if so, what happened.
4. The students are encouraged to work with partners to make observations about the information in the chart and their rectangular models. They should notice that the numbers in the first two columns of any row could be multiplied to get 24 (the area). The teacher may note their observations by writing " $L \times W = 24$ " and use the term factors of 24 as another way, in addition to length and width, to describe the numbers in the first two columns.
5. Some students may notice that as the numbers for one dimension increase, those for the other dimension decrease. Still others may note that the perimeters were always even. They may also consider the rectangles at the bottom of the chart were the same as the ones at the top, just turned different ways. **This observation may prompt the teacher to remind the students that they have talked about this idea as a property of multiplication—the commutative property—and as congruence of figures.**
6. Ask the students to describe the rectangles with the greatest and smallest perimeters. Students will observe that the long "skinny" rectangles had greater perimeters than the "fatter" rectangles. The teacher models this by taking the 1-unit-by-24-unit rectangle of perimeter 50, split it in half, and connect the halves to form the 2-unit-by-12-unit rectangle. As the teacher moves the tiles she/he explains that some tile edges on the outside boundary of the skinny rectangle were moved to the inside of the wider rectangle. Because there were fewer edges on the outside, the perimeter of the rectangle decreases.

Names _____

24 TILES

Commutative Property

Length (L) Units	Width (W) Units	Area (A) (sq. units)	Perimeter (P) (units)

Describe any relationships that you discovered:

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CONTENT STANDARDS

Primary Standard

Strand Name: Algebra

SD Goal: Students will use the language of algebra to explore, describe, represent, and analyze number expressions and relations that represent variable quantities.

Indicator: Use procedures to transform algebraic expressions.

Standard: 4.A.1.2 Students are able to recognize and use the commutative property of addition and multiplication.

Supplemental Standard

Strand Name: Geometry

SD Goal: Students will use the language of geometry to discover, analyze, and communicate geometric concepts, properties, and relationships.

Indicator: Use properties of geometric figures to solve problems from a variety of perspectives.

Standard: 4.G.2.1 Students are able to compare geometric figures using size, shape, orientation, congruence and similarity.

NCTM Process Standard

Problem Solving

- Build new mathematical knowledge through problem solving
- Apply and adapt a variety of appropriate strategies to solve problems

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics

Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Use the language of mathematics to express mathematical ideas precisely

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Representation

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems

Problem-Solving Strategies

- Modeling
- Looking for patterns
- Use of manipulatives

ASSESSMENT TOOLS

Chamberlain Elementary Schools

Math Rubric



Name: _____

Teacher: Mrs. Ford

Date Submitted: _____

Title of Work: _____

	Criteria				Points
	4	3	2	1	
Explanation	A complete response with a detailed explanation.	Good solid response with clear explanation.	Explanation is unclear.	Misses key points.	_____
Use Of Visuals	Complete chart with correct answers.	Clear chart.	Incomplete or unclear chart.	No chart.	_____
Mechanics	No math errors.	No major math errors or serious flaws in reasoning.	May be some serious math errors or flaws in reasoning.	Major math errors or serious flaws in reasoning.	_____
Demonstrated Knowledge	Shows complete understanding of the questions, mathematical ideas, and processes.	Shows substantial understanding of the problem, ideas, and processes.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	_____
Requirements	Goes beyond the requirements of the problem.	Meets the requirements of the problem.	Hardly meets the requirements of the problem.	Does not meet the requirements of the problem.	_____
				Total---->	_____

**Fourth Grade Algebra
Performance Descriptors**

Advanced	Fourth grade students performing at the advanced level: <ul style="list-style-type: none"> • solve algebraic equations using inverse operations and order of operations with addition and subtraction using whole numbers; • solve word problems by converting them to algebraic statements; • create patterns to solve problems and justify their solution.
Proficient	Fourth grade students performing at the proficient level: <ul style="list-style-type: none"> • use the commutative property of addition and multiplication; • identify and complete patterns and describe the associated rule; • write and solve number sentences using whole numbers; • simplify a two-step equation using whole numbers; • show relationships between all operations; • simplify whole number expressions in all operations; • select appropriate relational symbols to make number sentences true.
Basic	Fourth grade student performing at the basic level: <ul style="list-style-type: none"> • show relationship between addition and subtraction; • simplify whole number expressions in addition and subtraction; • using whole numbers, solve number sentences.

**Fourth Grade Algebra
ELL Performance Descriptors**

Proficient	Fourth grade ELL students performing at the proficient level: <ul style="list-style-type: none"> • write and solve number sentences that represent word problems; • use variables as place holders in number sentences; • recognize simple patterns; • identify and complete patterns and describe the associated rule; • read, write, and speak the language of mathematics.
Intermediate	Fourth grade ELL students performing at the intermediate level: <ul style="list-style-type: none"> • solve simple number sentences using the four basic operations and a model; • create numerical expressions from oral or written contexts; • explain in mathematical terms the sequence of steps in solving two-step problems; • give simple oral or written responses to directed questions on topics presented in class.
Basic	Fourth grade ELL students performing at the basic level: <ul style="list-style-type: none"> • write numerals and mathematics symbols; • solve problems using addition, subtraction, and multiplication; • recognize and use basic algebraic terms; • respond to yes or no questions and to problems presented pictorially or numerically in class.
Emergent	Fourth grade ELL students performing at the emergent level: <ul style="list-style-type: none"> • begin to use number sentences using symbolic representations; • give simple oral responses to directed questions on topics presented in class; • copy and write numerals and mathematics symbols; • imitate pronunciation of numbers and mathematical terms; • use non-verbal communication to express mathematical ideas.
Pre-emergent	Fourth grade ELL students performing at the pre-emergent level: <ul style="list-style-type: none"> • observe and model appropriate cultural and learning behaviors from peers and adults; • listen to and observe comprehensible instruction and communicate understanding non-verbally.

24 TILES

Student Work Samples



As you examine the samples, consider the following questions:

- In light of the standard/s addressed and the assessment tools provided, what evidence does the work provide that students are achieving proficiency in the knowledge and skills addressed by the standard/s for the task?
- Is the task/activity well designed to help students acquire knowledge and demonstrate proficiency? Is the task/activity clearly aligned with the standards? In what ways would you adapt the task/activity to better meet the needs of your students?

Student Work Sample #1

Name _____

Length (L) (units)	Width (W) (units)	Area (A) (sq. units)	Perimeter (P) (units)
8	3	24	22
2	12	24	24
3	8	24	22
12	2	24	24
6	1	24	20
4	6	24	20
1	24	24	50
24	1	24	50

Describe any relationships that you discovered.

If you turned them you will get a different length and width but you will get the same area and perimeter.

Looking at Student Work – Instructor notes and rating for work sample #1

Chamberlain Elementary Schools Math Rubric



Name: _____

Teacher: Mrs. Ford

Date Submitted: 1-17-05

Title of Work: Commutative Property

	Criteria				Points
	4	3	2	1	
Explanation	A complete response with a detailed explanation.	Good solid response with clear explanation.	Explanation is unclear.	Misses key points.	<u>4</u>
Use Of Visuals	Complete chart with correct answers.	Clear chart.	Incomplete or unclear chart.	No chart.	<u>4</u>
Mechanics	No math errors.	No major math errors or serious flaws in reasoning.	May be some serious math errors or flaws in reasoning.	Major math errors or serious flaws in reasoning.	<u>4</u>
Demonstrated Knowledge	Shows complete understanding of the questions, mathematical ideas, and processes.	Shows substantial understanding of the problem, ideas, and processes.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	<u>4</u>
Requirements	Goes beyond the requirements of the problem.	Meets the requirements of the problem.	Hardly meets the requirements of the problem.	Does not meet the requirements of the problem.	<u>3</u>
				Total-->	<u> </u>

Teacher Comments: *Advanced- This work was given an advanced score because the student clearly understood the commutative property. His statement confirmed his understanding.*

Student Work Sample #2

Length (L) (units)	Width (W) (units)	Area (A) (sq. units)	Perimeter (P) (units)
12	2	24	28
6	4	24	20
3	8	24	22
2	12	24	28
4	6	24	20
8	3	24	22
24	1	24	50
1	24	24	50

Describe any relationships that you discovered. You could only do four different ones that could work. So you had to switch those four around to make eight.

Looking at Student Work – Instructor notes and rating for work sample #2

Chamberlain Elementary Schools Math Rubric



Name: _____

Teacher: Mrs. Ford

Date Submitted: 1-17-05

Title of Work: Commutative Property

	Criteria				Points
	4	3	2	1	
Explanation	A complete response with a detailed explanation.	Good solid response with clear explanation.	Explanation is unclear.	Misses key points.	<u>3</u>
Use Of Visuals	Complete chart with correct answers.	Clear chart.	Incomplete or unclear chart.	No chart.	<u>4</u>
Mechanics	No math errors.	No major math errors or serious flaws in reasoning.	May be some serious math errors or flaws in reasoning.	Major math errors or serious flaws in reasoning.	<u>4</u>
Demonstrated Knowledge	Shows complete understanding of the questions, mathematical ideas, and processes.	Shows substantial understanding of the problem, ideas, and processes.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	<u>3</u>
Requirements	Goes beyond the requirements of the problem.	Meets the requirements of the problem.	Hardly meets the requirements of the problem.	Does not meet the requirements of the problem.	<u>3</u>
				Total---->	<u>17</u>

Teacher Comments: *Proficient - This work was given a proficient score because the student correctly completed the chart and had a solid response using the word "switch".*

Student Work Sample #3

Name _____

Length (L) (units)	Width (W) (units)	Area (A) (sq. units)	Perimeter (P) (units)
8	3	$8 \times 3 = 24$	18 ✓
12	2	$12 \times 2 = 24$	24
4	6	$4 \times 6 = 24$	16
3	8	$8 \times 8 = 24$	18 ✓
2	12	$2 \times 12 = 24$	24

Describe any relationships that you discovered.

It would always equal 24 for the area.

Looking at Student Work – Instructor notes and rating for work sample #3

Chamberlain Elementary Schools Math Rubric



Name: _____

Teacher: Mrs. Ford

Date Submitted: 1-17-05

Title of Work: Commutative Property

	Criteria				Points
	4	3	2	1	
Explanation	A complete response with a detailed explanation.	Good solid response with clear explanation.	Explanation is unclear.	Misses key points.	<u>2</u>
Use Of Visuals	Complete chart with correct answers.	Clear chart.	Incomplete or unclear chart.	No chart.	<u>3</u>
Mechanics	No math errors.	No major math errors or serious flaws in reasoning.	May be some serious math errors or flaws in reasoning.	Major math errors or serious flaws in reasoning.	<u>3</u>
Demonstrated Knowledge	Shows complete understanding of the questions, mathematical ideas, and processes.	Shows substantial understanding of the problem, ideas, and processes.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	<u>2</u>
Requirements	Goes beyond the requirements of the problem.	Meets the requirements of the problem.	Hardly meets the requirements of the problem.	Does not meet the requirements of the problem.	<u>3</u>
				Total---->	<u>13</u>

Teacher Comments: *Basic - This work was given a basic score because the student the relationship of (2×12) and (12×2) always equal to 24. The perimeter was incorrect on two problems.*

INSTRUCTIONAL NOTES

Author Comments

I presented the lesson, and then let the students manipulate the tiles and discover combinations to complete the chart. After they completed their charts we had a whole class discussion about area, perimeter, and commutative property. Following the discussion, students worked with a partner to repeat the task and discuss their results. There seemed to be increased understanding the second time.

Resources

SD Mathematics Content Standards

<http://www.doe.sd.gov/contentstandards/math/index.asp>

SD Assessment and Testing

<http://www.doe.sd.gov/octa/assessment/index.asp>

The National Assessment of Educational Progress (NAEP)

<http://www.doe.sd.gov/octa/assessment/naep/index.asp>

National Council of Teachers of Mathematics

<http://nctm.org/>

Looking at Student Work

<http://www.lasw.org/index.html>